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Electricity market equilibrium model with seasonal volatilities

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Abstract

In this paper we propose and implement an electricity market equilibrium model. The model, originally conceived by Hinz [8], is now set up by making use as input of the spot price pattern obtained with the term structure Heath Jarrow Morton [6] model, but we assume that price volatility is seasonal. The chosen volatility functional form captures the price return seasonality and consequently allows to support the production scheduling. We show as seasonality and electricity demand forecasting techniques make the study of energy forward price dynamics related to the demand and the provisional capacity of the agents.

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1. Introduction

The electricity market deregulation, started in the United States and then diffused in the main western countries, has determined new price dynamics development. The electricity market, once monopolistic, becomes a competitive market, where energy prices derive from the demand/supply match. This new context, along with the physical characteristics of the electricity, has generated new price dynamics, never seen before, nor on the financial markets or commodities markets. The fundamental characteristics of such good is not to be stored: with the exception of

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hydropower, electricity should be generated exactly at the time of its request for consumption. The offer turns out to be completely inelastic to price changes, and also prices show very high volatilities and sudden changes of price levels, called spikes.

Market operators, both producers and consumers, suffer therefore the exposure to energy price uncertainty and risk management techniques become fundamental tools to quantify and cope with price risk generated from such uncertainty.

For this reason both researchers and practitioners have concentrated on the electricity price evolution study, by setting up models able to catch the main characteristics of price evolution in order to price derivative products and manage price risk.

Electricity is a flow commodity: all contracts guarantee the delivery of a certain amount of energy (1 MWh) continuously over a time interval (1 hour, 1 month, 1 quarter, 1 year). Electricity is traded on an auction system with standardized contracts, which can be settled both with physical delivery and financially.

In the last few years the electricity market and other flow commodities stochastic models literature has rapidly developed. In particular two research lines have been created: the traditional methodology is build up on the spot price stochastic process modelling, by adapting the original approach already used with other flow commodities.

Lucia and Schwarz [11] model the natural logarithm of the spot price by assuming a mean reverting process estimated by using the spot price data in the Nordic market. The price evolution of a future contract is than determined by applying expected value under an appropriate martingale measure equivalent to the objective one.

Other authors ([13] e [4]) suggest a two factor model, in order to take into account the influence on the spot price given both by a short term and a long term source of randomness. The introduction of a jump diffusion process appears the natural way to account for spikes ([4] e [2]), even if market incompleteness is introduced. Huisman [9], Deng [3] and Mari [12] among others, suggested Markovian regime switching models characterized by the occurrence of stable and turbolent periods.

The major disadvantage of the spot price models is that forward prices are given endogenously from the spot price dynamics. Therefore, the obtained dynamics of forward prices is most of the times not consistent with the market observed prices.

The second research line refers to the modeling framework of Heath Jarrow and Morton [6] that, using only few stochastic factors and the initial price curve as given, models futures prices under some equivalent martingale measure in a no-arbitrage environment. Clewlow and Strickland [2] have been the first researchers to introduce this approach to the energy market. Bjerksund et al. [1] and Koekebakker et al. [10] model a continuum of instantaneous-delivery forward contracts under risk neutral probability measure.

To tell the truth it has never been explicitly made clear how it is possible to apply the term structure classical models to an anomalous market such the electricity one: the non storability of electrical power cannot be neglected using a price methodology in which the underlying is assumed storable. Besides, hedging is impossible if short position are not allowed and the electricity market is far away from being complete, so it is not guaranteed the existence of a risk neutral measure.

Hinz [8] gives an interesting interpretation of the electricity market and demonstrates that it is possible to create a market framework where it is guaranteed the existence of a risk neutral measure: the energy cannot be stored, but it can be produced. And the producer can put himself in the condition of having the ability to produce electricity, creating a sort of "electricity storability". According to this perspective, the electricity market becomes more complex and has to be considered as composed of both power electricity and agreements on power production capacities. The market reaches the equilibrium and determines the price process for all tradable assets both physical (production capacity agreements) and financial (future electricity prices). The equilibrium existence gives and economic interpretation of the martingale measure Q: it is equivalent to the market measure P, such that equilibrium asset prices under P are given by their future revenues, expected with respect to Q. Price dynamics can be, therefore, described directly under the equivalent martingale measure Q and it becomes possible to price all contracts by using classical no arbitrage models of financial mathematics.

According to this approach Fanelli and Musti [5] have obtained the forward price evolution in the electricity market by applying the HJM term structure model to the Currency Change Electricity Market. It has been possible to consider various volatility levels according to the year months, reflecting the electricity price seasonality.

In this paper, having as input the spot prices obtained with the methodology introduced by Fanelli Musti [5], we numerically implement the Hinz [7] equilibrium model. In paragraph 2 we describe the equilibrium model we are implementing in its single-period and multi-period version.

In paragraph 3 we expose the methodology to obtain the spot price series as input of the equilibrium model. Results and conclusions are exposed in paragraph 4.

2. Equilibrium model

We consider at this stage the one period version of the model. The market is composed by N agents acting simultaneously as retailer as well as producers of electricity.

In the single period model there are two dates: T_0 represents today and T_1 represents tomorrow, the delivery time. At time T_0 each agent doesn't know the energy request he will be asked for the following day T_1. This amount is going to represent an exogenous random variable.

The agent energy request can be satisfied both by purchasing today on the day ahead market with price p and by tomorrow production, by using his own power plants. The choice about how to satisfy the request depends on the economic valuation based on prices and quantities. The profit and loss function for agent *i* is the following ([7]):

$$G_i\left(p, q_i, \widetilde{Q}_i, \widetilde{p}^s\right) = p_i^r \widetilde{Q}_i + p^b\left(q_i - \widetilde{Q}_i\right)_+ - P_i^f - pq_i - \min\left\{p_i^\upsilon, \widetilde{p}^s\right\}\min\left\{\left(\widetilde{Q}_i - q_i\right)_+, c_i\right\} - \widetilde{p}^s\left(\widetilde{Q}_i - q_i - c_i\right)_+$$

where:

p	(variable) is 1 MWh day-ahead price
q_i	(control variable for the agent) is the amount of energy purchased today by the agent on the day- ahead market
$\widetilde{Q}_i$	(variable) is the amount of energy that the agent i will be asked for at time T_1
$\widetilde{p}^s$	(variable) is the tomorrow 1 MWh spot price
p_i^r	(fixed) is the selling price the agent i will obtain by selling 1 MWh of electricity
p^b	(fixed) is the "back supply price", price obtained selling 1 MWh (for the production exceeding the request)
P_i^f	(fixed) is the general cost of production for 1 MWh of energy
p_i^υ	(fixed) is the variable cost of production for 1 MWh of energy
c_i	(fixed) is the production capacity for agent

Each agent at time T_0 establishes the quantity of energy to buy on the day-ahead market and the interaction among decisions of the N agents determines the price p. The random variables are represented by the quantities $\widetilde{Q}_i$ and $\widetilde{p}^s$. The profit and loss function is concave respect to the control variable q_i and it guarantees the optimal solution for every agent maximizing his expected utility

$$E\left[U_i\left(G_i\left(p, q_i, \cdot, \cdot\right)\right)\right]$$

subject to the global production capacity constrains, so that every agent can sell at least his production, buy at least the all system production and the net balance of all quantities has to be null:

$$q_i \in \left[-c_i, \sum_{j \neq i} c_j\right]$$

$$\sum_{i=1}^{N} q_i = 0 .$$

Hinz in [7] demonstrates that the equilibrium price exists if spot price is greater that variable cost, even if the uniqueness of the equilibrium point is still to be investigated.
The multi period model represents the natural extension of the single period model, and the random variables already introduced are labeled with the temporal notation and represented as a $(N+1)$-dimension stochastic process:

$$\tilde{\pi}(n) = \left(\tilde{p}^s(n), \tilde{Q}_1(n), \tilde{Q}_2(n), \ldots, \tilde{Q}_N(n)\right)_{n \geq 1} \qquad (1)$$

where $\tilde{p}^s(n)$ represents the spot price of 1 KWh at time n, while $\tilde{Q}_i(n)$ represents the KWh quantity of energy requested by agent i at time n.
The filtration generated by the process, $F^{\tilde{\pi}}$, represents the information generated by the process and an equilibrium is defined as a process adapted to the filtration $F^{\tilde{\pi}}$

$$\left(p^*, q_1^*(n), q_2^*(n), \ldots, q_N^*(n)\right)_{n \geq 1}$$

such that any couple $\left(p^*, q_i^*(n)\right)$ maximizes *i*-th agent expected utility at time $n \geq 1$, given the information available at the former date $(n-1)$.

3. Numerical implementation

The model is implemented in its multi-period version and in the hypothesis we formulate there are no differences among the agents, so that they sustain the same costs and have the same probability distribution of the energy request. With these hypothesis the equilibrium price is the following([7]):

$$p^*(n) = p^\upsilon - E\left[\left(p^\upsilon - \tilde{p}^s(n+1)\right)_+\right] + E\left[\left(\tilde{p}^s(n+1) - p^\upsilon\right)_+\right] P\left(\tilde{Q}(n+1) \geq c \,\middle|\, \tilde{\pi}(n)\right). \qquad (2)$$

The series

$$\tilde{a}_n = p^\upsilon - E\left[\left(p^\upsilon - \tilde{p}^s(n+1)\right)_+\right] \qquad (3)$$

$$\widetilde{b}_n = p^{\upsilon} - E\left[\left(p^{\upsilon} - \widetilde{p}^{s}(n+1)\right)_+\right] + E\left[\left(\widetilde{p}^{s}(n+1) - p^{\upsilon}\right)_+\right] \tag{4}$$

represent the equilibrium price variability interval.

The series $\widetilde{a}_n$ and $\widetilde{b}_n$ are identified once we observe the spot price realization $\widetilde{p}_n$ and we can forecast the possible evolution at the following time $(n+1)$.

In order to numerically implement the model we have to make hypothesis regarding the spot price $\widetilde{p}_s$ and demand $\widetilde{Q}_i$ dynamics.

The first fundamental input to implement the equilibrium model is the spot price series $\widetilde{p}^s(n)$, obtained by applying the model Fanelli et al. [5]. We consider realistic the use of a price evolution taking into account the price seasonality, by the monthly calibration of volatility parameters. Indeed, we consider as input the spot price resulting from simulations in a Currency Change Electricity Market (CCEM): prices are determined by applying, in a seasonal volatility regime, the Heath Jarrow Morton [6] model. The currency change represents the technical instrument to make the HJM model suitable in the electricity market (Hinz [7]) .

The HJM model features make it highly versatile because of the forward rate volatility function choice: in particular it is possible to take into account the seasonality in price volatility and in price level. In particular we consider a path dependent volatility that captures the seasonality feature of forward rates, with monthly regime switching volatility. The price process is obtained by simulating the daily entire forward curve stochastic evolution over 1 year period and its corresponding spot price evolution $\widetilde{p}_s$.

The second fundamental input to implement the equilibrium model is the demand process. The process is given by the discrete sequence of random variables

$$\widetilde{Q}_i(n+1) = \mu\, e^{\alpha x(n-1) + \beta \eta(n)}$$

where $\mu, \alpha, \beta \in \Re^+$, $\eta(n)$ is, for each n , a standard normal variable and $x(n)$ is the ARMA(2,1) process

$$x(n+1) = \varphi_1 x(n) + \varphi_2 x(n-1) + \vartheta_1 \varepsilon(n) + \vartheta_2 \varepsilon(n+1)$$

$$|\varphi_i| < 1, \quad |\vartheta_i| < 1, \quad i = 1,2\,, \quad \varepsilon(n) \sim \mathrm{N}(0,1)$$

driving the electricity demand addressed to each agent in the market.

We suppose that the electricity requests received by the agents have the same probability distribution, and $\widetilde{Q}(n)$ is the random variable representing the electricity demand addressed to each agent.

We simulate the demand path and the corresponding conditional probabilities:

$$\mathrm{P}\left(\widetilde{\mathrm{Q}}(\mathrm{n}+1) \geq c \,\middle|\, \widetilde{\pi}(n)\right).$$

Conditional probabilities are obtained referring to a fixed production level capacity c of the agent, and the demand path is simulated according to the following parameter values:

Table 1

Parameter	Value
φ_1	0.95
φ_2	-0.7
ϑ_1	0.2
ϑ_2	0.2
α	1.5
β	1.0
μ	1.0
c	1.5

The condition $c = 1.5 > \mu$ indicates that a large part of demand is met with in-house production rather than by purchasing electricity on the market.
Finally, the equilibrium price (2) is obtained by simulating the electricity spot price paths with daily discretization over one year time and Montecarlo method. In Figure 1 is presented the equilibrium price path: the price results to vary between the series (3) and (4), representing therefore the variability boundaries of the equilibrium price.
Finally, the agent capability of forecasting demand levels for the following day becomes crucial in order to define the equilibrium price on the day ahead market. This ability is deeply influenced by the demand volatility and it is possible to take it into account simply by fixing parameter $\beta > \mu$. Indeed, how we can notice in figure 2, the price path P1 has been obtained with the hypothesis of low predictability

$$0.1 = \beta < \mu = 0.5,$$

whilst the equilibrium price path P2 has been obtained by assuming agents good provisional capacity level

$$0.1 = \mu < \beta = 0.5.$$

In our opinion it is interesting to notice how the high predictability of demand level, determines lower equilibrium prices: as shown in Figure 2, the price path P2 is most of the time close to the minimum boundary a_n.
On the other hand, low predictability level causes high volatile prices showing that agents accept to pay prices closer o the maximum boundary b_n.
The equilibrium model gives the opportunity to take into account the seasonality of the two components, electricity prices and electricity demand. The calibration of the volatility function in the HJM model and of the ARMA model for the demand evolution, represent the key instruments to take the physical characteristics of the electricity into the model implementation.

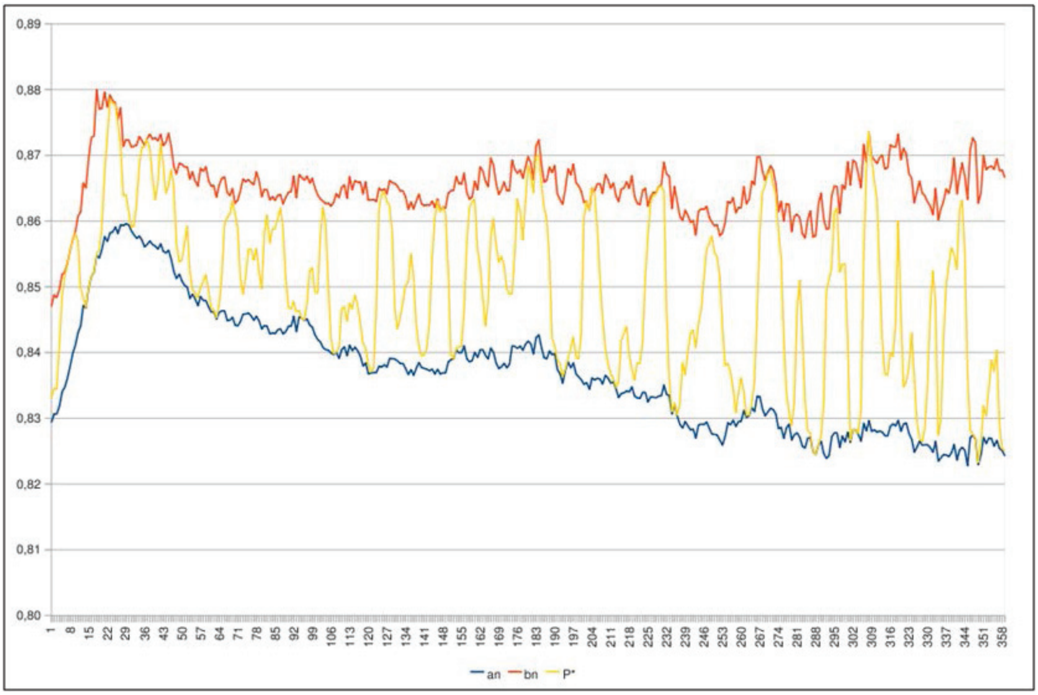

Figure 1. Equilibrium prices

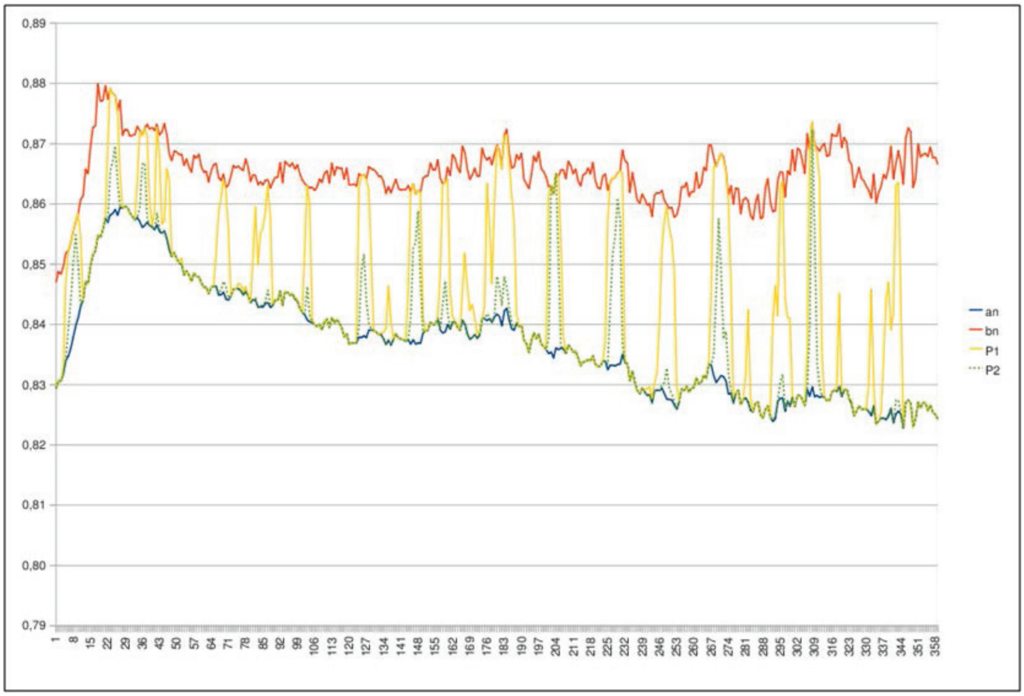

Figure 2. Equilibrium prices with hypothesis of different demand predictability.

4. Conclusions

In the electricity market, seasonality is without doubt one of the most important feature of the price dynamics. The proposed equilibrium model and its implementation allow us to take into account the seasonality of electricity price variability by simply calibrating the seasonal volatility as input of the model.

The spot price path is determined by taking into account the historical volatilities observed in the market and therefore it allows to simulate in a realistic way the spot price evolution, once the demand evolution is given as input. The demand is supposed to be driven by an ARMA (2,1) process.

The seasonality in the volatility function driving the spot price $\widetilde{p}_s$ could be easily introduced even in the demand dynamic, by studying the equilibrium price in case of demand driven by a moving average integrated auto regressive model, such as seasonal ARIMA process. We consider this natural extension as the opportunity for further research.

Furthermore, Hinz [8] is the first researcher dealing with the use of classical no arbitrage models in the energy market price modeling: the term structure model appears therefore coherent in order to determine a spot price as input to obtain the price in the equilibrium model. The implementation gives the opportunity to evaluate different hypotheses about demand process; the possibility to study the interaction between price/demand could be, in our opinion, greatly valuable for the production scheduling in the electricity market. The model implementation points out how the agent capability of forecasting demand level plays an important role in the equilibrium price definition. Indeed, in the high predictability case, lower price levels reveal a better market efficiency and resources allocation.

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